



# Word Statistics

1/26/2026

# Today (and next class)

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- Word-level metrics, statistics, Bayesian Inference
- Exploratory text analysis
  - First approaches when working with a new data set – what can we do with minimal supervision? Minimal information about the data?

# Background

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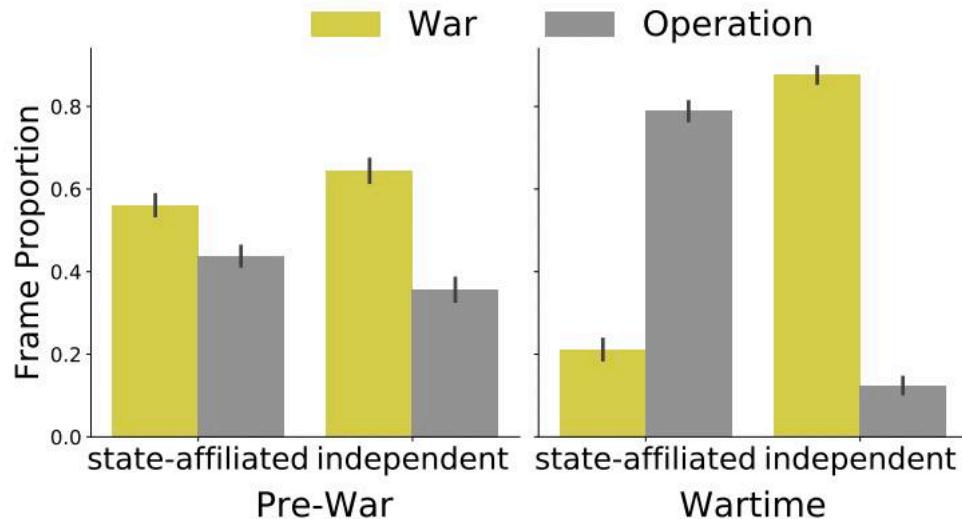
- One of the most fundamental analyses we may want to conduct is, how does word usage differ in different corpora?
  - How do AI policy discussions differ in the U.S. and Europe?
    - Maybe U.S. politicians use words like “innovation” while European politicians use words like “privacy” [fictional example]
  - How do Wikipedia articles about men and women differ?
    - Articles about women focus on family and relationships more than articles about men (Wagner et al. 2015) [fictional words: “family”, “children”, “married”, “divorce”]
- “Entries in the burgeoning “text-as-data” movement are often accompanied by lists or visualizations of how word (or other lexical feature) usage differs across some pair or set of documents”

# Example: Russia-Ukraine War

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# Example: State-affiliated outlets use “operation” over “war”



- We know to look for these terms because of laws passed in Russia, but what if we want to discover these differences?

# Running Example: Congressional Record

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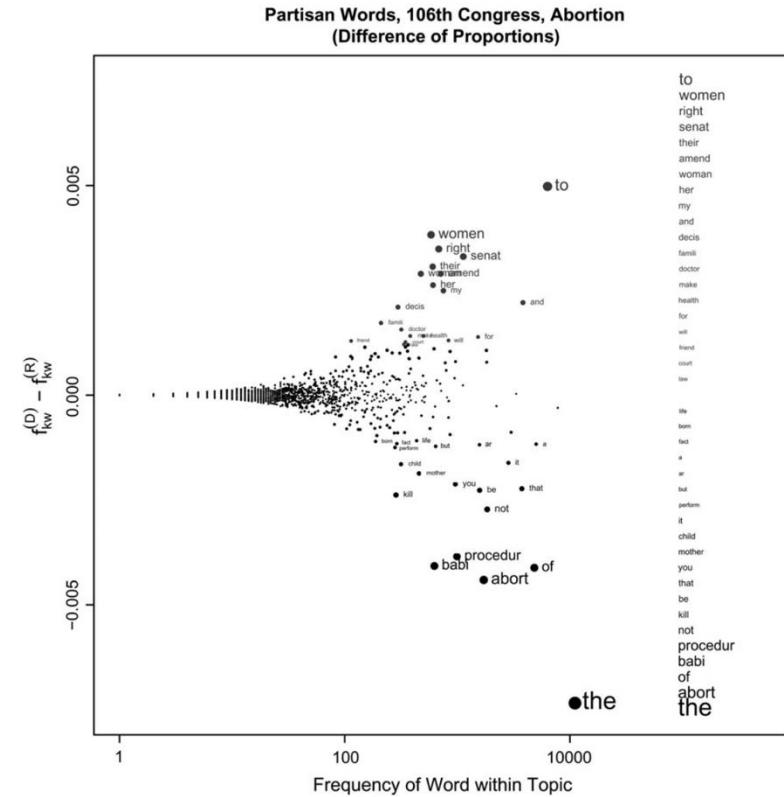
- How does word usage differ in speeches made by Republican and Democratic members of congress?
  - “The question is not which of these terms are partisan and which are not, but which are the most partisan, on which side, and by how much.” [Monroe et al. 2008]

Data credits:

- The corpus was originally constructed in plaintext format by Gentzkow, Shapiro, and Taddy (2018) ([repository for full download](#), [license](#)).
- Preprocessed by Rodriguez and Spirling (2021) ([code](#), [R data file](#)): remove non-alphabetic characters, lowercase, and remove words of length 2 or less, then filter to Congressional sessions 111-114 (Jan 2009 - Jan 2017) and to speakers with party labels D and R.
- Converted plaintext and csv files and subsampled by [Sandeep Soni](#) and [Connor Gilroy](#) ([code](#))

# Some initial ideas: proportion of words

- Which words have the highest frequency in statements by Democrats?
  - "the", "and", "that", "this", "for", "have", "are", "not"
- Which words have the highest frequency in statements by Republicans?
  - "the", "and", "that", "for", "this", "have", "are", "our"



# Some initial ideas: Odds ratio

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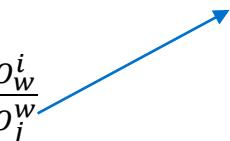
- Odds ratio:  $O_w^{(i)} = \frac{f_w}{1 - f_w}$ , where  $f_w$  is the proportion of word  $w$  in corpus subset  $i$
- Odds ratio between two groups:  $\theta_w^{(i-j)} = \frac{O_w^i}{O_j^w}$
- Log-odds ratio:  $\log(O_w^i) - \log(O_j^w)$   is symmetrical

# Some initial ideas: Odds ratio

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- Odds ratio:  $\theta_w^{(i)} = \frac{f_w}{1 - f_w}$
- Odds ratio between two groups:  $\theta_w^{(i-j)} = \frac{\theta_w^i}{\theta_w^j}$
- Log-odds ratio:  $\log(\theta_w^i) - \log(\theta_w^j)$

Becomes infinite/undefined if words only exist in one corpus



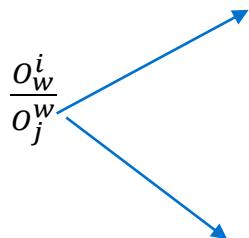
# Odds ratio in Congressional data

Word	Odds-Ratio	Frequency in Republican Speech	Frequency in Democratic Speech
idahoans	-5.46	211	1
fairtax	-4.99	131	1
cdh	-4.75	103	1
isna	-4.71	99	1
zinser	4.96	1	161
gaspee	4.74	1	128
vermonters	4.59	5	555
corinthian	4.57	2	218

# Some initial ideas: Odds ratio

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- Odds ratio:  $\theta_w^{(i)} = \frac{f_w}{1 - f_w}$
- Odds ratio between two groups:  $\theta_w^{(i-j)} = \frac{\theta_w^i}{\theta_j^w}$
- Log-odds ratio:  $\log(\theta_w^i) - \log(\theta_j^w)$

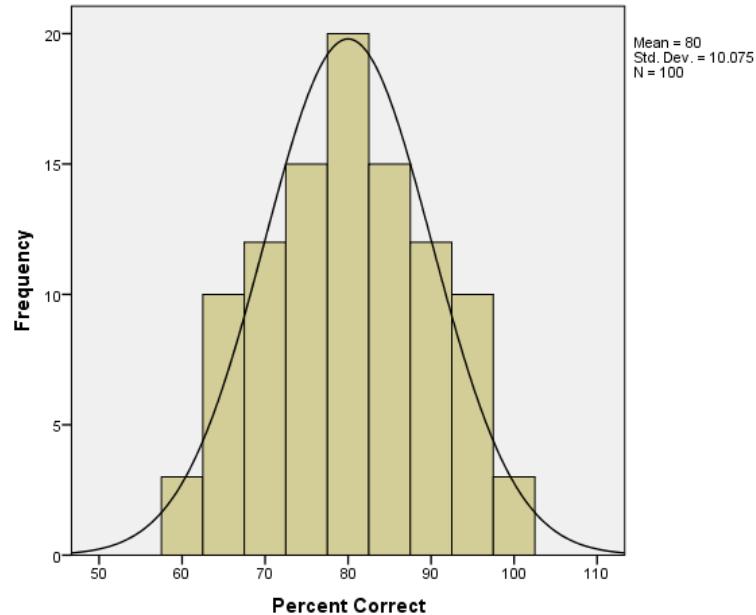


Becomes infinite if words only exist in one corpus

Becomes dominated by obscure words

# Model-driven approach

- Clear that simple methods aren't going to work
- General statistical modeling approach:
  - Given a collection of data
    - Assume you generated this data from some model
    - Estimate model parameters
- Example:
  - Assume you gathered data by sampling from a normal distribution
  - Estimate mean and stdev



# Model-driven approach

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- High-level idea:
  - First model the word usage in the full collection of documents
  - Then investigate how subgroup-specific word usage diverges from that in the full collection of documents
- Incorporate a *prior*
  - Background estimate of how often a word is used in this type of document

# Bag-of-words (BOW) assumption

- "the state of healthcare in this country is..."
- We ignore ordering of words and assume that we can represent the document collection as a "bag of words"
- [We've already been doing this implicitly]

country state the  
in this  
healthcare  
is of

# Terminology

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- **y** = vector of term counts in the corpus

<b>101</b>	<b>60</b>	<b>10</b>	...	<b>11</b>	<b>231</b>
<b>country</b>	<b>state</b>	<b>healthcare</b>	...	<b>employment</b>	<b>the</b>

- $n$  = number of terms in the corpus
- $n = 101 + 60 + 10 \dots + 11 + 231$

# Terminology

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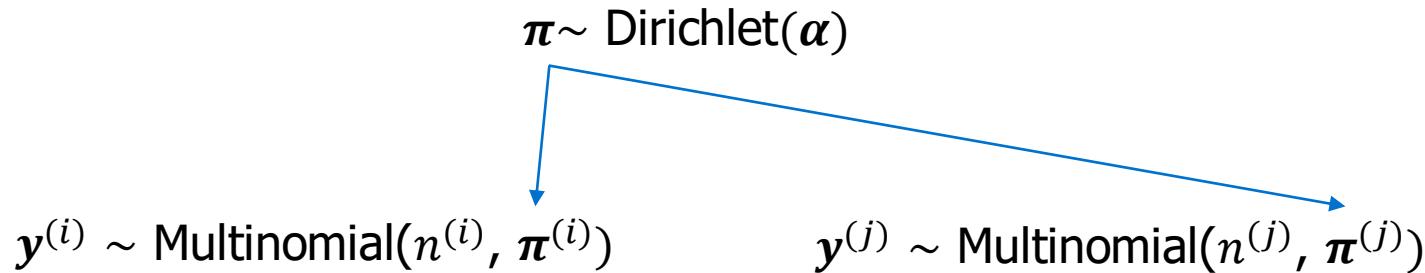
## Define:

- $\mathbf{y}$  = vector of term counts in the corpus
- $n$  = number of terms in the corpus
- $\pi$  = unknown distribution the vocabulary

## ▪ Assume:

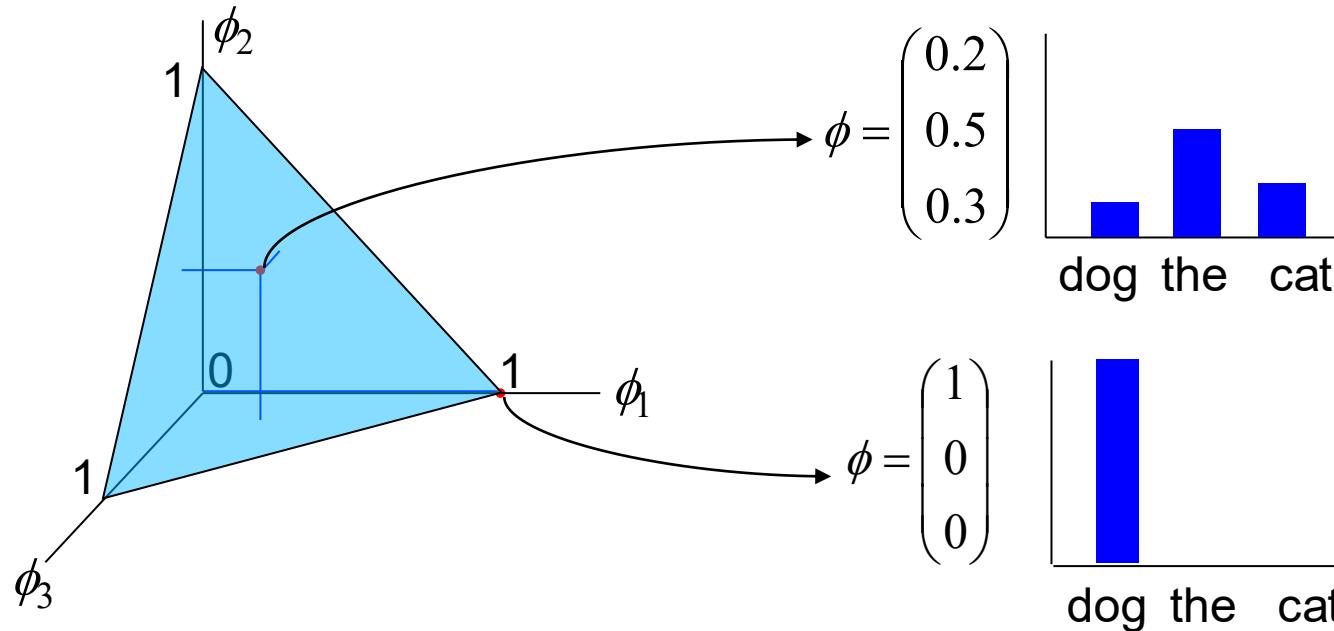
- $\mathbf{y} \sim \text{Multinomial}(n, \pi)$
- Intuition: we got  $\mathbf{y}$  by repeatedly sampling from a bag.  $\pi$  describes how many of each word is in the bag

# Impose *Dirichlet Prior* on $\pi$



# What is a Dirichlet distribution?

- We can plot multinomial probability distributions
- Shape we get is a *simplex*

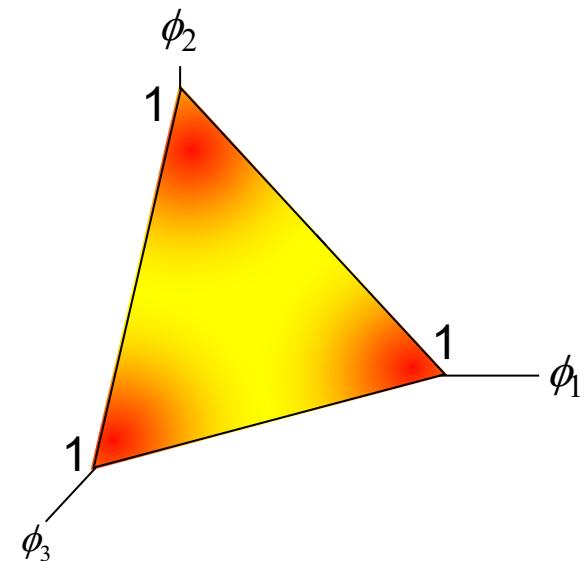
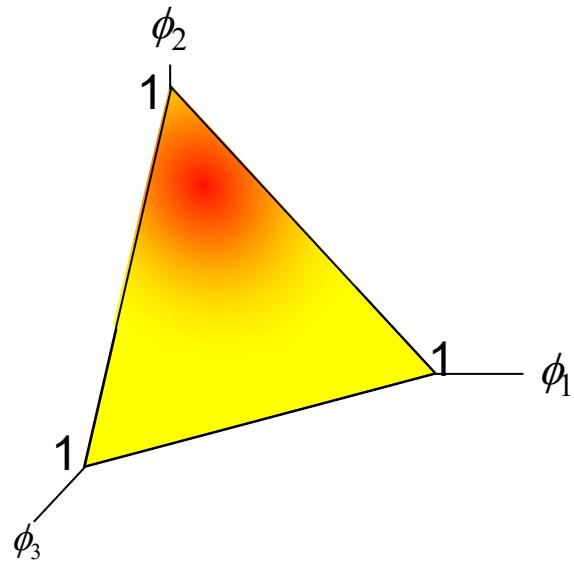
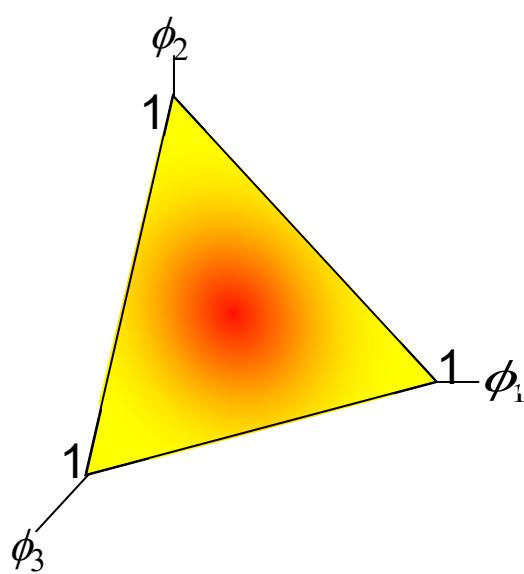


$$\sum_i \phi_i = 1$$

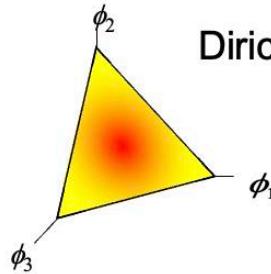
$$\sum_i \phi_i = 1$$

# What is a Dirichlet distribution?

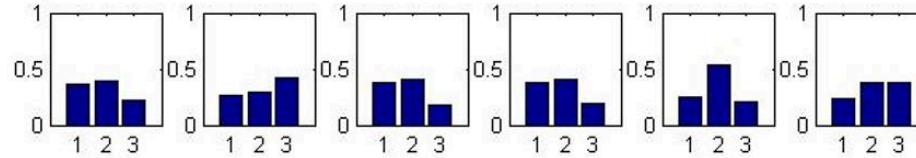
- A Dirichlet distribution is a distribution over multinomial distributions  $\phi$  in the simplex



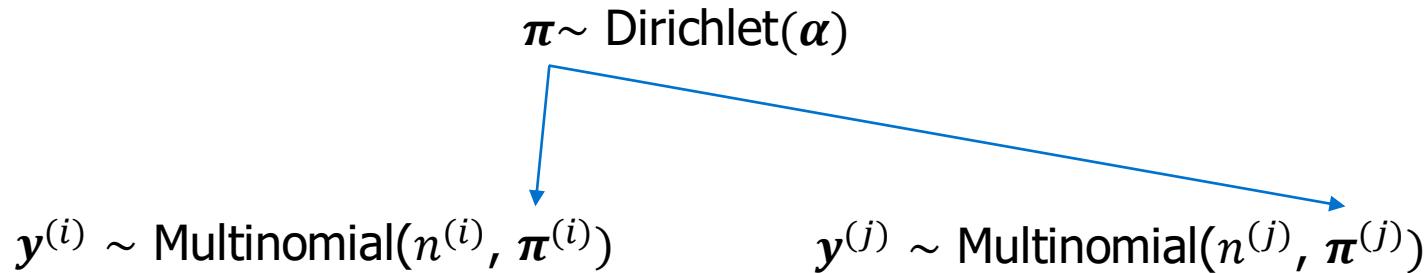
# Example draws from a Dirichlet distribution over the 3-simplex



Dirichlet(5,5,5) [higher alpha – more dense]

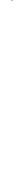


# Impose *Dirichlet Prior* on $\pi$



# Impose *Dirichlet Prior* on $\pi$

$$\pi \sim \text{Dirichlet}(\alpha)$$



Frequency of a term in  
the *entire* corpus

321	176	53	...	54	543
country	state	healthcare	...	employment	the

# Impose *Dirichlet Prior* on $\pi$

$$\begin{array}{ccc} \pi \sim \text{Dirichlet}(\alpha) & & \\ \downarrow & & \searrow \\ y^{(i)} \sim \text{Multinomial}(n^{(i)}, \pi^{(i)}) & & y^{(j)} \sim \text{Multinomial}(n^{(j)}, \pi^{(j)}) \end{array}$$

$y^{(i)}$  can be word frequencies for Democrat Speech  
 $y^{(j)}$  can be word frequencies for Republican Speech

Both are assumed to have the same prior – frequency in general congressional speech

# Generative Story

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1. Draw  $\pi^{(i)} \sim \text{Dirichlet}(\alpha)$
2. For  $n^{(i)}$  steps:
  1. Draw  $w \sim \text{Multinomial}(\pi^{(i)})$

For each subset of our corpus,

- $y^{(i)}$ ,  $n^{(i)}$  and  $\alpha$  are observed in the data (where  $y^{(i)}$  contains counts of  $w$ )
- $\pi^{(i)}$  is what we need to estimate

# Another aside about distributions

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- Prior distribution:  $P(\pi)$
- Posterior distribution:  $P(\pi | w)$
- When the posterior distribution is in the same family as the prior distribution, they are called **conjugate distributions**
- The Dirichlet distribution is a **conjugate prior** of the multinomial distribution
- [For our purposes, we often chose a Dirichlet prior for a multinomial distribution because it makes inference easier]

# Point estimate of $\pi$

$$\hat{\pi}_w^{(i)} = \frac{y_w^{(i)} + \alpha_w}{n^{(i)} + \alpha_0} \quad \longrightarrow \text{Point estimate of } \pi, \text{ where } \alpha_0 = \sum \alpha_w$$

Intuitive interpretation: imagine we saw  $\alpha_0$  additional words and  $\alpha_w$  were  $w$

# Point estimate of $\pi$

$$\hat{\pi}_w^{(i)} = \frac{y_w^{(i)} + \alpha_w}{n^{(i)} + \alpha_0} \longrightarrow \text{Point estimate of } \pi, \text{ where } \alpha_0 = \sum \alpha_w$$

$$\hat{\delta}_w^{(i-j)} = \log\left(\frac{\pi_w^{(i)}}{1-\pi_w^{(i)}}\right) - \log\left(\frac{\pi_w^{(j)}}{1-\pi_w^{(j)}}\right) \longrightarrow \text{Log-odds ratio with } \pi \text{ instead of frequencies}$$

$$\hat{\delta}_w^{(i-j)} = \log\left(\frac{y_w^{(i)} + \alpha_w}{n^{(i)} + \alpha_0 - y_w^{(i)} - \alpha_w}\right) - \log\left(\frac{y_w^{(j)} + \alpha_w}{n^{(j)} + \alpha_0 - y_w^{(j)} - \alpha_w}\right)$$

# Congressional data with Dirichlet prior

Word	$\delta$ (rounded)	Frequency in Republican Speech	Frequency in Democratic Speech
idahoans	-0.7321	210	0
fairtax	-0.7321	130	0
cdh	-0.7321	102	0
isna	-0.7321	98	0
zinser	0.6542	0	160
gaspee	0.6542	0	127
vania	0.6542	0	105
fiveminute	0.6542	0	95

We don't have to drop zero counts anymore, but this isn't that much better than before!

We could impose a stronger prior?

# Variance

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- Report *z-score*: point estimate divided by variance
  - Lower-frequency words have higher variance

With some assumptions, we can estimate:

$$\sigma^2(\hat{\delta}_w^{(i-j)}) \approx \frac{1}{y_w^{(i)} + \alpha_w^{(i)}} + \frac{1}{y_w^{(j)} + \alpha_w^{(j)}}$$

And use as our final score:

$$\frac{\delta_w^{(i-j)}}{\sqrt{\sigma^2(\delta_w^{(i-j)})}}$$

# Odds ratio in Congressional data

Top Republican Words	Score	Top Democrat Words	Score
spending	-66.26	republican	56.63
obamacare	-59.90	wealthiest	40.78
government	-47.92	rhode	39.43
going	-45.33	women	38.16
that	-44.58	pollution	33.66
trillion	-43.43	republicans	32.86
taxes	-42.39	gun	32.45
you	-40.85	investments	32.22
administration	-39.07	families	31.93
debt	-38.92	violence	30.88

# New Example: Narrative framing in restaurant reviews

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- As online reviews have become commonplace, they offer an opportunity to study consumer behavior
- How do consumers frame positive and negative sentiment online?
- Data:
  - 900,000 Yelp restaurant reviews from 9 cities: Boston, Chicago, Los Angeles, New York, Philadelphia, San Francisco, and Washington D.C
  - Corpus subsets:
    - “i” = one star reviews
    - “j” = 5 star reviews
    - Prior: entire review corpus

# New Example: Narrative framing in restaurant reviews

Table 2: Top 50 words associated with one-star reviews by the Monroe, *et al.* (2008) method.

Linguistic class	Words in class
<b>Negative sentiment</b>	worst, rude, terrible, horrible, bad, awful, disgusting, bland, tasteless, gross, mediocre, overpriced, worse, poor
<b>Linguistic negation</b>	no, not
<b>First person plural pronouns</b>	we, us, our
<b>Third person pronouns</b>	she, he, her, him
<b>Past tense verbs</b>	was, were, asked, told, said, did, charged, waited, left, took
<b>Narrative sequencers</b>	after, then
<b>Common nouns</b>	manager, waitress, waiter, customer, customers, attitude, waste, poisoning, money, bill, minutes
<b>Irrealis modals</b>	would, should
<b>Infinitives and complementizers</b>	to, that

"In summary, one-star reviews were overwhelmingly focused on narrating experiences of trauma rather than discussing food, both portraying the author as a victim and using first person plural to express solace in community."

# More serious example: Racial differences CPS services

- Words used in caseworker notes about families referred to child protective services
- Compare words used in notes about about Black families vs. white families

Black-assoc.	Score	White-assoc.	Score
<b>Referrals</b>			
she	52.19	he	54.64
belt	47.37	heroin	41.87
her	45.39	PGF	36.08
BM	37.90	treatment	36.16
bus	30.95	anxiety	34.25
shelter	25.11	using	27.45
whooped	23.96	therapist	26.05
<b>Cases</b>			
school	56.80	F	130.67
housing	42.01	parents	59.26
informed	37.76	drug	37.65
pass	35.75	methadone	36.55

# Alternate Approach: Pointwise-mutual information

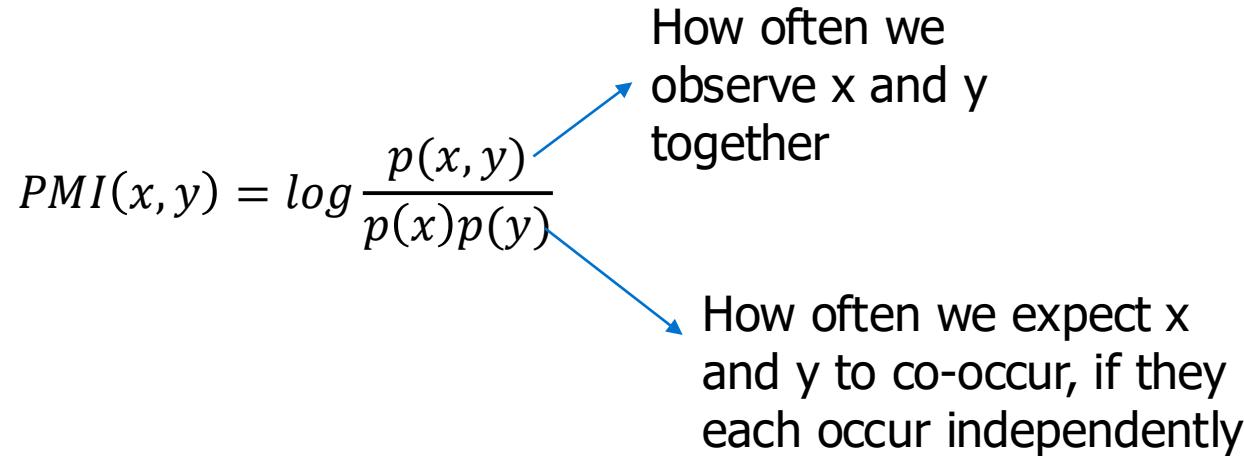
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- Probability/Information theory measure of association
- Common formulation: measure how often two events, x and y occur, compare with what we would expect if they were independent

$$PMI(x, y) = \log \frac{p(x, y)}{p(x)p(y)}$$

How often we observe x and y together

How often we expect x and y to co-occur, if they each occur independently



# Alternate Approach: Pointwise-mutual information

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- Compute the co-occurrence between a word  $w$  and a label  $i$

$$PMI(w, i) = \log \frac{p(w, i)}{p(w)p(i)}$$

Probability of  $w$  and  $i$  co-occurring

Probability of  $w$  occurring

Probability of  $i$  occurring

# Computing PMI

$$PMI(w, i) = \log \frac{p(w, i)}{p(w)p(i)} = \log \frac{p(w|i)p(i)}{p(w)p(i)} = \log \frac{p(w|i)}{p(w)}$$

	<b>country</b>	<b>state</b>	<b>healthcare</b>	<b>...</b>	<b>employment</b>	<b>the</b>	<b>Total</b>
Republican	321	176	15	...	54	500	10233
Democratic	100	31	53	...	20	543	12231
<b>Total</b>	421	207	68	...	74	1043	22464

$$PMI(\text{Republican}, \text{employment}) = \log \frac{\left(\frac{54}{22464}\right)}{\left(\frac{74}{22464}\right)\left(\frac{10233}{22464}\right)}$$

# Alternate Approach: Pointwise-mutual information

---

- Compute the co-occurrence between a word  $w$  and a label  $i$

$$PMI(w, i) = \log \frac{p(w, i)}{p(w)p(i)}$$

Number of times  $w$  occurs in  $i$ -labeled documents / number of total words

Proportion of  $w$

Proportion of  $i$ -labeled terms

# Alternate Approach: Pointwise-mutual information

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- Common to use *Positive Pointwise mutual information (PPMI)*
  - Set PMI to 0 wherever it is negative
- Still run into problems with over-emphasizing rare words:
  - There are some fixes for this, including smoothing
- PMI scores are used frequently

# Example of PMI: Gender Bias on Wikipedia

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- [Only include words that occur in at least 1% of biographies]
- Women: actress (15.9%), women's (8.8%), female (5.6%), **her husband** (4.1%), women (5.3%), first woman (1.9%), film actress (1.6%), her mother (1.8%), woman (4.4%), **nee** (3.6%), feminist (1%), miss (1.9%), model (3.3%), girls (1.5%) and singer (6.5%).
- Men: played (14.2%), footballer who (3.0%), football (4.5%), league (5.9%), john (7.9%), major league (1.8%), football league (1.6%), college football (1.5%), son (7%), football player (2.2%), footballer (2%), served (11.7%), william (4.6%), national football (2%) and professional footballer (1%).

# Additional Applications

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- PPMI and variants of odds ratio are commonly used as *features* in other NLP tasks (not just for word statistics on their own)
  - Represent a document using one of these metrics instead of using word counts
  - Document vectors can be used for similarity metrics, e.g. clustering or information retrieval

	<b>country</b>	<b>state</b>	<b>healthcare</b>	...	<b>employment</b>	<b>the</b>
Republican	321	176	15	...	54	500
Democratic	100	31	53	...	20	543

# Today's takeaways

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- Counting words can be surprisingly hard!
- Key ideas behind two popular methods for examining word statistics:
  - Log-odds with a Dirichlet prior ("Fightin' Words")
  - Pointwise mutual information scores
- Examples of applications and understanding of when these methods are useful

# Some takeaways from the course goals from

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- Range of places people are interested in applying concepts from this course: public health, economics, cognitive science, environmental science
- Mix of background in NLP
- Topics of interest: misinformation, bias/inequality, validation, LLMs

# Reminders

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- Course website: <http://nlp-css-601-672.cs.jhu.edu/sp2026/>
- Join class Piazza
- Fill out course goals survey (linked on slides from last class)

# References

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- Jurafsky and Martin, 2022, Sec 6.6
  - [https://web.stanford.edu/~jurafsky/slp3/ed3book\\_jan122022.pdf](https://web.stanford.edu/~jurafsky/slp3/ed3book_jan122022.pdf)
- Monroe BL, Colaresi MP, Quinn KM. Fightin' Words: Lexical Feature Selection and Evaluation for Identifying the Content of Political Conflict. *Political Analysis*. 2008;16(4):372-403. doi:10.1093/pan/mpn018

# End

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